The special nontopological scalar solitons in anti de Sitter spacetimes

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Abstract

In this letter the nontopological scalar solitons are investigated in an anti de Sitter spacetime. We find analytically that the solitons obeying the necessary conditions $m = \frac{2}{\sqrt{3}}|\Lambda|^{\frac{1}{2}}$ or $m = \sqrt{\frac{2(n+3)(2n+3)}{3}}|\Lambda|^{\frac{1}{2}}$ can exist in the background by means of the series expansion.

KEY WORDS: nontopological soliton, anti de Sitter spacetime

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1. Introduction

Recently more attention is paid to the anti de Sitter (AdS) spacetime due to the mentioned corredpondence between physical effects by gravitating fields propagating in AdS spacetime and those of a conformal field theory on the boundary of the above spacetime [1, 2]. The AdS spacetime can be obtained by compactifying the string theory, which helps us to consider the relevant problems deeply [2]. It is necessary to investigate a lot of topics and models within the cosmological surrounding with negative curvature.

Researching nontopological solitons (NTS) is valuable and also attracts more attention. Many kinds of stable NTS are cosmologically significant. It was shown that Q-balls, a kind of NTS, can become promising candidates for collisional dark matter and the model of NTS can be created copiously during the early Universe [3]. The NTS with positive self-interacting potential simulate the halo and provide a qualitatively better fit to the rotation curves [4]. A generalized Q-stars including a complex scalar field and Goldstone field may be considered as an elegant candidate for diffuse gaseous clouds in the universe and Einstein ring observed with microarcsecond X-ray imaging mission [5].

It is important to explore fields within the framework of AdS spacetimes with negative cosmological constant because a lot of string-inspired theories work mainly in the backgrounds. Some soliton stars such as Q-stars have been studied in AdS spacetimes and many interesting results were obtained [6]. As solutions to the field equation for NTS, they must satisfy the boundary conditions [7]. It is essential to discuss the models in AdS spacetimes in detail, which has a great influence upon the related topics in the same environment.

In this letter we derive the differential equations describing the NTS in AdS spacetimes. Under the boundary conditions, we discuss the solution to the equations of motion with series expansion. It is found that only some of NTS can exist in AdS spacetimes, depending on the spacetime structure. The results are finally emphasized.

Now we choose the metric of four-dimensional anti de Sitter spacetime as,

$$ds^2 = (1 + \frac{r^2}{l^2})dt^2 - \frac{dr^2}{1 + \frac{r^2}{l^2}} - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{1}$$

where $\Lambda = -\frac{3}{l^2}$. The Lagrangian for NTS is,

$$\mathcal{L} = \sqrt{-g}(\partial_\mu \Phi \partial^\mu \Phi^* - U(\Phi\Phi^*)) \tag{2}$$

where

$$U(\Phi\Phi^*) = m^2\Phi\Phi^* + \sigma(\Phi\Phi^*)^2 + \lambda(\Phi\Phi^*)^3 \tag{3}$$

the potential with minimum $U_{min}(\Phi\Phi^*) = 0$ at $\Phi = 0$. The equation of motion reads,

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\partial^\mu \Phi) + U(\Phi\Phi^*) = 0 \tag{4}$$

The complex scalar fields vary harmonically with time and the ansatz with frequency ω is,

$$\Phi(t,r) = P(r)e^{-i\omega t} \tag{5}$$

where r is radial coordinate. By means of ansatz (5), the field equation is reduced to,

$$(1 + \frac{r^2}{l^2})\frac{d^2P}{dr^2} + 2(\frac{1}{r} + \frac{2r}{l^2})\frac{dP}{dr} + \frac{\omega^2}{1 + \frac{r^2}{l^2}}P - m^2P - 2\sigma P^3 - 3\lambda P^5 = 0 \tag{6}$$

Introducing the transformation,

$$\frac{r}{l} = \frac{x}{\sqrt{1 - x^2}} \tag{7}$$

then equation (6) becomes,

$$(1 - x^2)^2\frac{d^2P}{dx^2} + (1 - x^2)(\frac{2}{x} - x)\frac{dP}{dx} + \omega^2 l^2(1 - x^2)P - m^2 l^2 P - 2\sigma l^2 P^3 - 3\lambda l^2 P^5 = 0 \tag{8}$$

According to transformation (7), it is obvious that $\frac{r}{l} \in [0, \infty)$ is equivalent to $x \in [0, 1)$. As fields for NTS which obey the necessary conditions, they vanish at infinity so as to lead the potential (3) be zero there. The required boundary conditions are that $P(x = 0)$ is finite and $P(x \rightarrow 1) = 0$.

There exists a Noether current for our model,

$$j^\mu = \sqrt{-g}g^{\mu\nu}i(\Phi^*\partial_\nu\Phi - \Phi\partial_\nu\Phi^*) \tag{9}$$

The current is conserved,

$$j^\mu_{;\mu} = 0 \tag{10}$$

The total charge is denoted as,

$$Q = \int d^3x j^0 \tag{11}$$

and can be taken as,

$$Q = 8\pi \int \frac{\omega P^2}{1 + \frac{r^2}{l^2}}r^2 dr \tag{12}$$

We solve equation (8) explicitly with the series in the form,

$$P(x) = \sum_{n=0}^{\infty} a_{2n}x^{2n} \tag{13}$$

Within the region $x \in [0, 1)$, the recursion formula of the coefficients a_k can be expressed as follow,

$$a_0 = P(x = 0) \tag{14}$$

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$$a_2 = \frac{a_0 l^2}{6}[3\lambda a_0^4 + 2\sigma a_0^2 + (m^2 - \omega^2)] \tag{15}$$

$$
\begin{aligned}
a_{2n+4} = \frac{1}{2(n+2)(2n+5)}\Big\{ & [2(n+1)(4n+5) + (m^2 - \omega^2)l^2]a_{2n+2} \\
& + (\omega^2 l^2 - 4n^2)a_{2n} + 2\sigma l^2 \sum_{i+j+k=2n+2} a_i a_j a_k \\
& + 3\lambda l^2 \sum_{i+j+k+p+q=2n+2} a_i a_j a_k a_p a_q \Big\}
\end{aligned} \tag{16}
$$

where $i, j, k, p, q = 0, 2, 4, \cdots$, $n = 0, 1, 2, \cdots$. On the other hand, within the asymptotic region near $x = 1$ or approaching the infinity, the solution to equation (8) can be chosen in the form of series,

$$P(x) = \sum_{n=0}^{\infty} b_n (1-x)^n \tag{17}$$

The coefficients b_k are found to be,

$$(3\lambda b_0^4 + 2\sigma b_0^2 + m^2)b_0 = 0 \tag{18}$$

$$b_1 = \frac{2\omega^2 + m^2 + 2\sigma b_0^2 + 3\lambda b_0^4}{2 + m^2 l^2 + 6\sigma l^2 b_0^2 + 15\lambda l^2 b_0^4} l^2 b_0 \tag{19}$$

$$(4 - m^2 l^2)b_2 = (3 - 2\omega^2 l^2 - m^2 l^2)b_1 + 3\omega^2 l^2 b_0 \tag{20}$$

$$
\begin{aligned}
b_{n+3} = \frac{1}{2(n+3)(2n+3) - m^2 l^2}\Big\{ & [(n+2)(8n+11) - 2\omega^2 l^2 - m^2 l^2]b_{n+2} \\
& + [3\omega^2 l^2 - (n+1)(5n+4)]b_{n+1} + (n^2 - \omega^2 l^2)b_n \\
& + 2\sigma l^2 \sum_{k+l+m=n+3} b_k b_l b_m - 2\sigma l^2 \sum_{k+l+m=n+2} b_k b_l b_m \\
& + 3\lambda l^2 \sum_{i+j+k+p+q=n+3} b_i b_j b_k b_p b_q - 3\lambda l^2 \sum_{i+j+k+p+q=n+2} b_i b_j b_k b_p b_q \Big\}
\end{aligned} \tag{21}
$$

From expressions (18-21), the first several coefficients determine the others. According to the boundary conditions mentioned above, we must choose $b_0 = 0$, then $b_1 = 0$ from expression (19). If all of b_n $(n = 0, 1, 2, \cdots)$ are equal to zero, we will obtain the trivial solution $P(x \le 1) = 0$ from (17). In the whole space, there can not exist the nontopological solitons, the smooth and nontrivial solutions according to (13) and (17). In order to avoid the trivial solution, we choose $ml = 2$ from expression (20), then the coefficient b_2 can be chosen as nonzero. The other coefficients b_k $(k > 2)$ are obtained by means of equation (21). If $ml \ne 2$, then $b_2 = 0$. We can also keep some of other coefficients nonzero if we let the denominator in (21) vanish. For $m^2 l^2 = 2(n+3)(2n+3)$, some of coefficients $b_j = 0$ $(0 \le j \le n+2)$ and the others b_k $(k \ge n+3)$ can not vanish, where $n = 0, 1, 2, \cdots$.

The NTS imposed with $m = \frac{2}{\sqrt{3}}\sqrt{|\Lambda|}$ or $m = \sqrt{\frac{2(n+3)(2n+3)}{3}}|\Lambda|^{\frac{1}{2}}$ ($n = 0, 1, 2, \cdots$) can survive in the AdS spacetimes. The structures of spacetime limit the model. In addition, the potential (3) can be generalized. It can possess higher power terms and keep the position of its minimum. The generalized potential will not change our conclusions because only m, the coefficient of $\Phi\Phi^*$-term, is limited.

We discuss the equation of motion (8) numerically. The solutions are depicted in the Figure with $ml = 2$. Here we let $\sigma = -1$, $\lambda = 1$ for simplicity.

It is indicated that not all NTS that form in the flat spacetimes can live in the AdS. Only some of them with restriction $m = \frac{2}{\sqrt{3}}\sqrt{|\Lambda|}$ or $m = \sqrt{\frac{2(n+3)(2n+3)}{3}}|\Lambda|^{\frac{1}{2}}$ ($n = 0, 1, 2, \cdots$) can inhabit in the cosmological surrounding. The interesting results encourage us to continue exploring the NTS in the different spacetimes.

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Figure 1: The solid, dadot, dashed curves for $P(x)$ in AdS spacetime with $\Lambda = -0.1, -0.25, -0.3$ respectively.